The q-Tensor Square of a Powerful p-Group, $q \geq 0$

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Abstract. Let p be a prime number and G a finite p-group. We say that G is powerful if $[G, G] \leqslant G^p$, for p odd, or if $[G, G] \leqslant G^4$, for $p = 2$. If N is a normal subgroup of G and satisfies $[N, G] \leqslant N^p$, for $p \geq 3$, or $[N, G] \leqslant N^4$, for $p = 2$, then N is said to be powerfully embedded em G. In this talk we consider the group $\nu^q(G)$, q a non-negative integer, as described for instance by Bueno and Rocco in [1], which happens to be an extension of the q-tensor square $G \otimes^q G$ by $G \times G$. Our purpose is to address some results concerning $\nu^q(G)$ and $G \otimes^q G$, under the assumption that G is a powerful p-group, which generalize results for $q = 0$ due to Moravec. More specifically:

Theorem A. *Let G be a powerful, finite $p-$group and q a non-negative integer. Then*

(i) $\gamma_i(\nu^q(G))$ *is powerfully embedded in* $\nu^q(G)$*, for* $i \geq 2$*;*

(ii) $\nu^q(G)_i$ *is powerfully embedded in* $\nu^q(G)$*, for* $i \geq 1$*.*

Here, as usual $\gamma_i(\nu^q(G))$ (resp., $\nu^q(G)_i$) denotes the i-th term of the lower central series (resp., derived series) of $\nu^q(G)$.

Let $d(G)$ (resp., $\exp(G)$) denote the minimal number of generators (resp., the exponent) of the group G.

Theorem B. *Under the hypothesis of Theorem A, assume that* $\exp(G)$ *divides* q *and write* d *for* $d(G)$*. Then*

(i) $G \otimes^q G$ *is powerfully embedded in* $\nu^q(G)$*;*

(ii) $d(G \otimes^q G) \leq d(d + 1)$.

In contrast with the case $q = 0$, for $q \geq 1$ the q-tensor square $G \otimes^q G$ involves a certain subgroup $K \trianglelefteq \nu^q(G)$, which plays an important role in its structure.

Theorem C. *Let G be a powerful p-group. Then*

(i) $\exp([\nu^q(G), \nu^q(G)])$ *divides* $\exp(G)$*;*

(ii) $\exp(K)$ *divides* $\exp(G)$ *if p is odd or if* $4 \mid q$*;*

(iii) $\exp(K)$ *divides* $2\exp(G)$ *if* $p = 2$ *and* $4 \nmid q$*.*

* This is a joint work with Noraí Romeu Rocco.

References

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